Get To Know Crest Factor And Its Influence On The Design Of OFDM-Based Systems

This article is the third in a series of quarterly guest columns by Justin Panzer, manager of product marketing, Rohde & Schwarz. This installment is coauthored by Bernhard Kaehs, a senior development expert in the company's broadcasting division.

The exceptional performance of wireless communications systems and most recently television broadcasting is delivered in large measure by digital transmission technology and more specifically by higher-order modulation schemes such as Orthogonal Frequency Division Multiplexing (OFDM). However, they present major design challenges, since OFDM systems can have peak-to-average power ratios as high as 20:1, making it essential to carefully specify RF power components in the transmission chain to ensure they will not only handle such high voltage peaks, but do so without distortion or even breakdown. Crest factor, the ratio of peak to average or thermal power, is one of the most important parameters in making these decisions and if determined accurately (even though it can only be assessed statistically), can make the designer's task far easier. To understand the importance of crest factor in designing a system using OFDM, it helps to review its fundamental concepts and statistical background, as well as practical considerations to remember when creating a system design.

OFDM is used by virtually every new and proposed wireless and television broadcast system, and the crest factors it achieves are well above the values that occur in analog transmitters. For an individual transmitter, it is possible to limit the ratio of peak power to the average (thermal) output power. However, if several transmitters are interconnected, crest factor increases with the addition of each one and can reach more than 100 times the thermal power level. As a result, RF power components can no longer be specified solely on thermal criteria. Even though the most dramatic voltage peaks occur only rarely in statistical terms, they still must be considered in the design. This is especially true at higher power levels where the voltages associated with these power peaks can produce flashover and possibly even a standing arc somewhere in the transmission chain.

As seen in the August 12, 2008 edition of the RF Globalnet (www.rfglobalnet.com) newsletter.

Crest Factor of a Modulated RF Signal

A signal's crest factor is computed from the ratio of peak voltage (Û) to the RMS value U. For a sinusoidal signal, a crest factor of √2 or 3.01 dB is obtained using the ratio Û/U of √2. For wireless systems, a sinusoidal carrier signal is modulated by a baseband signal that contains the desired information. If the modulation causes a change in carrier amplitude, crest factor increases as well. The time variation of carrier amplitude is known as the "envelope" of modulated signals, and two different crest factors can be produced that differ by 3.01 dB depending on how the result was determined. Consequently, specifying crest factor alone is meaningless unless it is stated along with the approach used to obtain it.

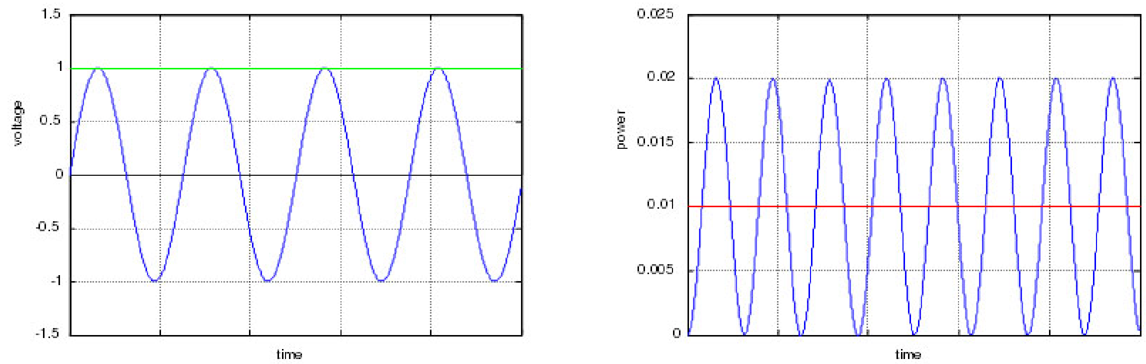
The first approach determines crest factor based on the highest amplitude peak that occurs in the modulated carrier signal and the RMS value. This article will refer to it as the "carrier approach" because it accounts for the RF carrier in addition to the envelope. This is important when specifying transmitter components because determining dielectric strength requires knowledge of the highest expected peak voltage.

The second way to determine crest factor uses the ratio of the modulation envelope's peak value to its RMS value, which will be called the "envelope approach". When compared to the carrier approach, it produces crest factor values that are reduced by the magnitude of the crest factor for the sinusoidal carrier (i.e. 3.01 dB). The technique is useful when evaluating an RF amplifier or a digital-to-analog converter operating at baseband.

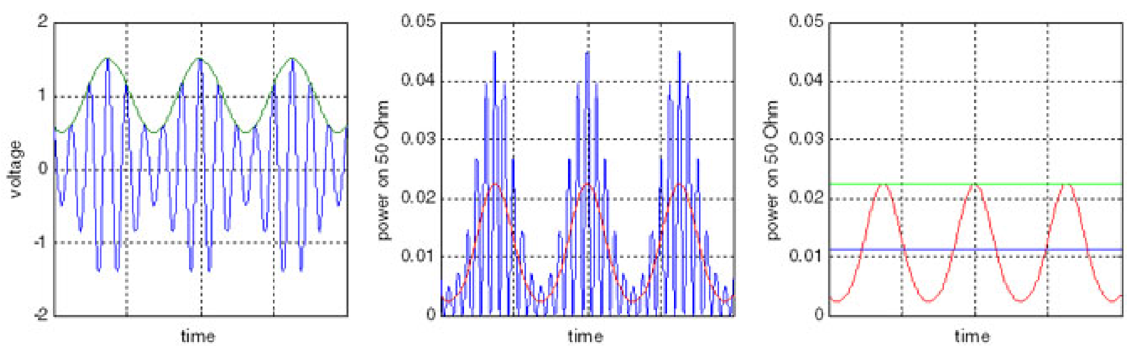
When expressed in decibels, crest factor as the ratio of peak voltage to RMS voltage can be analogous expressed as the ratio of peak envelope power to average power. With this approach, the crest factor resulting from the power ratio corresponds to the square value of the crest factor resulting from the voltage ratio. However, when indicated in dB, the two values are identical so it's not important to further differentiate between them.

References to peak-to-average ratio (PAR) are also found in the literature. When specified in dB, the PAR value corresponds to the crest factor obtained by using the carrier approach, so a sine wave carrier would have a PAR value of 3.01 dB. When dealing with modulated RF signals, "peak power" generally implies peak envelope power as the ratio of peak to average values and should be determined exclusively using the envelope approach.

The average power of a periodic signal is determined by squaring the voltage trace URF(t) and dividing it by the reference impedance. The value averaged over one period yields the signal's average power. For example, *Figure 1* shows a pure continuous wave (CW) signal. In *Figure 1a* the signal is shown with a constant envelope (green), and in *Figure 1b* as the squared trace (blue) referenced to 50 ohms along with the average power (red) that is constant for a CW signal. Power is determined in the same way as for a modulated signal, as shown in *Figure 2* using the same signal with amplitude modulation and a modulation depth of 0.5. *Figure 2a* shows the voltage trace URF(t) with a sinusoidal envelope. *Figure 2b* shows the squared time-domain signal referenced to 50 ohms (blue) with the power trace averaged over one period in red (which corresponds to the trace of the envelope power). The peak envelope power (*Figure 2c*) corresponds to the maximum value shown in green. The figure also shows the thermal power (blue) averaged over a long interval.



Figures 1a, 1b: *A pure continuous wave (CW) signal (blue) with a constant envelope (green) (a), the squared trace (blue) referenced to 50 ohms along with the average power (red) that is constant for a CW signal (b).*



Figures 2a, 2b, 2c: *The signal of Figure 1 with amplitude modulation and a modulation depth of 0.5. In (a) the voltage trace $U_{RF}(t)$ with a sinusoidal envelope is shown. In (b) the squared time-domain signal referenced to 50 ohms is shown in blue with the power trace averaged over one period in red. The peak envelope power (c) corresponds to the maximum value shown in green.*

Peak envelope power is the average power of the transmitter output during one period of the RF signal at the maximum value of the envelope, and is the value displayed by RF peak power meters. For a signal with a constant envelope, the peak envelope power is equal to the average power. The crest factor is thus equal to 1, which corresponds to 0 dB.

Crest Factor for Multiple Superimposed Signals

If several different transmit signals are switched to a single antenna, the signals will add vectorially, which increases crest factor. The same is true for an OFDM signal that consists of the superimposition of a number of individual modulated carriers. When several nonharmonic (i.e., uncorrelated) sinusoidal signals are superimposed, the amplitudes add up to produce a maximum total peak voltage. The signals are uncorrelated, so the power levels of the individual sinusoidal signals add to produce the average value of the sum signal (i.e., the squares of the RMS values of the voltages are added).

If two sinusoidal signals with the same amplitude are superimposed, the peak voltage will double. That is, the RMS value of the voltage will only increase by a factor of $\sqrt{2}$. So the crest factor Û/U will also increase by a factor of $\sqrt{2}$, i.e. by 3.01 dB. Using the carrier approach, crest factor is equal to 3.01 dB + 3.01 dB = 6.02 dB. This superimposition corresponds to a two-tone signal. Unlike the case of a single sinusoidal signal, the envelope is no longer constant. When the envelope approach is used, crest factor is reduced by 3.01 dB and is now equal to 3.01 dB (instead of 0 dB, which would be the case for a single sinusoidal signal). The approach also increases crest factor by 3.01 dB, which is caused by the superimposition of two signals, so the increase is independent of the approach being used. If the number of signals with the same amplitude is doubled, the crest factor increases by 3.01 dB.

For the crest factor of unmodulated and uncorrelated carriers with the same amplitude in dB, the following applies when using the envelope approach:

$$CF = 20 \cdot \log(\sqrt{n}) = 10 \cdot \log(n)$$

With the carrier approach, the following applies:

$$CF = 10 \cdot \log(n) + 3.01\text{ dB}$$

This correlation is also valid if the individual carriers are modulated and have a crest factor CFc (the crest factor of a single modulated carrier) (please define). For the crest factor of modulated and uncorrelated signals with the same amplitude in dB, the following applies regardless of the approach:

$$CF = 10 \cdot \log(n) + CFc$$

Measuring Crest Factor

With periodic signals, crest factor can be determined by measuring the peak and average or RMS values of the voltage or power using a power meter or oscilloscope. If the signal is measured for the duration of one period, the peak value and RMS value can be completely measured, assuming the test equipment is fast enough. This is not true for signals using a "random" modulation technique such as an OFDM. While the average value of a transmitter using OFDM can be determined precisely in only a few seconds using a thermal power meter, the magnitude of the peak value is highly dependent on measurement time. With high crest factors, signal peaks occur less frequently so it is necessary to specify the crest factor determined as well as the measurement interval or the number of samples.

It is important to understand the statistical probability of occurrence of signal peaks, and complementary cumulative distribution function (CCDF) offers a good starting point. CCDF is the statistical probability of occurrence of signal peaks that is greater by a factor k (in dB) than the average value. When generating an OFDM signal, an amplitude distribution arises in the baseband for the I and Q signals that is approximately normal in statistical terms. In other words, the I and Q signals resemble white noise. Based on the magnitude of the complex time-domain signal i(t) + jq(t), the envelope is formed during modulation onto an RF carrier and the amplitudes will have an approximate Rayleigh distribution.

The probability of occurrence of high signal peaks is significantly less with a Rayleigh distribution than with a normal distribution. However, if the carrier approach is used on an OFDM-modulated RF signal to statistically evaluate the amplitudes, a normal distribution will be obtained because of carrier oscillation. *Figure 3* shows a comparison of the CCDF for white noise (normal distribution) and for a Rayleigh distribution that occurs for the envelope of an ideal OFDM signal. The CCDF of a sinusoidal signal is also shown using the carrier approach. For a sinusoidal voltage, the peak value Û is greater than the RMS value U by a factor of √2. That is, the sinusoidal signal has a crest factor (Û/U) of 3.01 dB. No higher signal values occur, so their probability is 0.

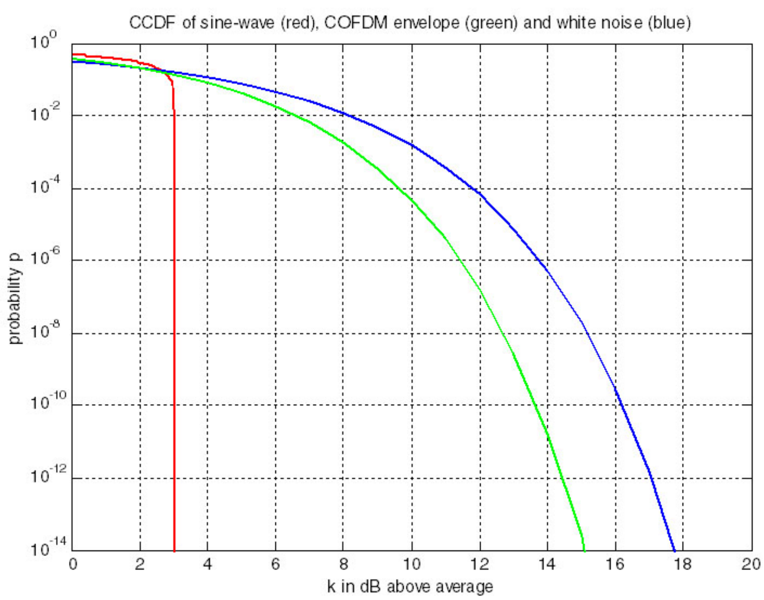


Figure 3: *CCDF for white noise (normal distribution) and for a Rayleigh distribution that occurs for the envelope of an ideal OFDM signal.*

In practice, crest factors of OFDM signals that exceed 12 dB using the envelope approach or 15 dB using the carrier approach are virtually impossible to measure even with the most sophisticated test equipment, which is illustrated in *Figure 3*. The probability of measuring signal levels that are only 1 dB higher is a factor of about 60 lower when using the envelope approach. In other words, measurement time must be multiplied by 60 to record signal peaks that are 13 dB above the average or RMS value. To record values at 14 dB, the measurement time must be multiplied by 10,000, and at 15 dB by 7 million. The signal duration of a power peak is about 110 ns for a DVB-T digital TV broadcast signal at 8 MHz (for example), so amplitude values that lie 12 dB over the average value occur on average once every second when using the envelope approach. At 15 dB, these peaks occur only once every 60 days. In other words, a signal peak that is 12 dB over the average or RMS value occurs on average for every 7.6 million samples during signal analysis.

If these factors are not considered in amplitude probability, a measured value may seem realistic because it appears stable even if the measurement interval is increased by a factor of 2 or even 10. The measured value is then sometimes incorrectly assumed to be the crest factor. It is also important to ensure that the dynamic range of the instrument used to perform the measurement is sufficient and that no converter or IF amplifier limits the crest factor. In some instruments, this dynamic range is indicated with a "margin" value. Too narrow an IF bandwidth or an IF filter with skirts that are too steep can also impair the results. Since an OFDM signal is limited for an individual transmitter, measurements ranging from seconds to minutes will generally produce accurate results.

Summary

Systems using higher-modulation schemes such as those embodied in OFDM make it essential that every element of the transmission chain, from RF components through complete transmitter subsystems, be chosen to withstand their demanding signal conditions. As this discussion has hopefully shown, correct assessment of crest factor offers many benefits to designers of these systems. Time taken early in the design process to determine the highest peak-to-average signal values an amplifier or transmitter will be required to endure will go a long way toward delivering an end product that not only “looks good on paper” but performs satisfactorily in the field as well.

Authors’ note: Information for this article was excerpted from Rohde & Schwarz Application Note 7TS02 entitled *The Crest Factor in DVB-T (OFDM) Transmitter Systems and Its’ Influence on Dimensioning RF Power Components*, which provides additional details about crest factor and its relationship to digital broadcast transmitters. It is available by clicking here.

About the authors

Justin Panzer is manager of product marketing for Rohde & Schwarz in North America. His background includes more than 14 years of test and measurement and mobile communications marketing experience. He has been with Rohde & Schwarz since 2003, with previous responsibility for mobile communications test products serving 2G and 3G technology markets. Panzer holds a B.S. in marketing from Drexel University and an MBA from Auburn University.

Bernhard Kaehs is a senior development expert in the Broadcasting Division of Rohde & Schwarz in Munich, Germany, which he joined in 1995. His background includes many years of experience in the design of solid-state VHF/UHF linear power amplifiers for TV applications in the kilowatt range. He holds a Dipl.-Ing.(FH) degree from the Munich University of Applied Sciences.